M5 Financial and Insurance Mathematics

Organiser: Jaakko Lehtomaa, *University of Helsinki*

1. Pathwise integrals and hedging

Lauri Viitasaari, *Aalto University*

In this talk we present some old and some recent results on the existence of pathwise stochastic integrals. These results together with the change of variable are used to show that with mild extra assumption on the driving process X that is Hölder continuous of order $\alpha > 1/2$, each random variable can be replicated with an adapted strategy. Consequently, with continuous trading one is able to hedge each financial instrument with zero cost leading to strong arbitrage in a model where the stock price is driven by the process X. This reveals once again the known fact that zero quadratic variation processes does not fit well to continuous financial models. Theoretical implications and applications are also discussed.

2. Mathematical models for longevity risk management

Helena Aro, *Etera mutual pension insurance company*

Longevity risk that stems from the continuous but uncertain improvements in general longevity poses significant challenges to the pensions and insurance industry as well as social security systems. This talk presents mathematical models for managing longevity risk. An overall objective is to develop methods for hedging cash flows of longevity-linked liabilities on financial markets. This is obtained by modelling the law of a multivariate stochastic process consisting of mortality and asset returns, with particular emphasis on the long-term development of mortality, and connections between mortality and asset returns. These connections are then utilized in designing hedging strategies for mortality-linked cash flows, using numerical methods.

More specifically, introduce a stochastic modelling framework for mortality, expressing the logistic transforms of annual cohort survival probabilities as a linear combination of user-specified basis functions of age. We then proceed to model the joint long-term development of mortality and financial markets, employing the mortality modelling framework. The resulting stochastic model is then applied to study optimal investment from the viewpoint of an investor with longevity-linked liabilities.

3. Post-optimal analysis of project portfolios in case of uncertainty

Vladimir Korotkov, *University of Turku*

Post-optimal analysis investigates the behaviour of the found solutions in response to the changes in the initial date. Some of the problems require the stability analysis of the found solutions when the initial data of the problem may contain errors or uncertainty. Even small perturbations in the initial data can lead to the situation when found optimal solution loses its optimality during the changes in the initial data. One of such problems with uncertainty in the initial data is the problem of the project portfolio selection. The initial data for this problem is obtained statistically and may be inaccurate. In the presentation it is outlined some results of the stability analyses for the project portfolio selection problem with several criteria.

There are presenting obtained quantitative characteristic of the stability of portfolios that show the upper level of changes in the initial date when the optimal portfolio remains optimal. Also the results related to speed of of losing the optimality of the portfolio is being presented.

4. Heavy-tailed distributions and ruin theory

Tanja Toivanen, *University of Helsinki / Ilmarinen Mutual Pension Insurance Company*

Heavy-tailed distributions play an important role in many fields while assessing the occurrence of a rare but potentially very harmful event. For instance, in insurance this might mean an exceedingly large claim and in dam building once in 10 000 years earthquake. In the talk some basic properties of heavy-tailed distributions are discussed. We will also see how a ruinous level is reached by a random walk consisting of random variables having heavy tails.