M11 Geometric and Functional Analysis

Organiser: Paola Elefante, *University of Helsinki*

1. Completion and extension of operators with a negative index in Kreĭn spaces

Dmytro Baidiuk, *University of Vaasa*

The famous results of M.G. Kreĭn concerning the description of selfadjoint contractive extensions of a Hermitian contraction T_1 and the characterization of all nonnegative selfadjoint extensions $\widehat{A}$ of a nonnegative operator A via the inequalities $A_K \leq \widehat{A} \leq A_F$, where A_K and A_F are the Kreĭn-von Neumann extension and the Friedrichs extension of A, are generalized to the situation, where $\widehat{A}$ is allowed to have a fixed number of negative eigenvalues. These generalizations are shown to be possible under a certain minimality condition on the negative index of the operators $I - T_1^* T_1$ and A, respectively; these conditions are automatically satisfied if T_1 is contractive or A is nonnegative, respectively.

In this talk a generalization of an old result due to Yu.L. Shmul'yan on completions of 2×2 nonnegative block operators will be explained. The extension of this fundamental result allows us to prove analogs of the above mentioned results of M.G. Kreĭn and, in addition, to solve some related lifting problems for J-contractive operators in Hilbert, Pontryagin and Kreĭn spaces in a simple manner.

2. INEQUALITIES AND BILIPSCHITZ CONDITIONS FOR TRIANGULAR RATIO METRIC

Parisa Hariri, *University of Turku*

Let $G \subsetneq \mathbb{R}^n$ be a domain and let d_1 and d_2 be two metrics on G. We compare the geometries defined by the two metrics to each other for several metrics such as the distance ratio metric, the triangular ratio metric and the visual angel metric. Finally we apply our results to study Lipschitz maps with respect to metrics.

References: [HVZ] P. Hariri, M. Vuorinen, and X. Zhang: Inequalities and bilipschitz conditions for triangular ratio metric. arXiv:1411.2747 [math.MG] 14pp.

3. Geometric Algebra and Analysis of Dirac Operators

Vesa Vuojamo, *Tampere University of Technology*

We introduce the Clifford algebras which can be considered as a general framework for generalizing complex numbers into higher dimensions. We present some basic properties of this construction. For instance rotations in space can be represented efficiently with the product of this algebra. The notion of complex analytic functions generalizes to Clifford algebras by using certain first-order operators often referred to as Dirac operators. We show some basic properties of these so-called monogenic functions and look into results analogous to standard complex analysis.